

BOUNDARY LAYER ANALYSIS OF MAGNETOHYDRODYNAMIC FLOW IN THE PRESENCE OF THERMAL RADIATION

Mehul Tithlia

Asst. Prof. (Mathematics)

Dolat –Usha Institute of Applied Sciences and Dhiru-Sarla Institute of Management & Commerce,
Valsad, Gujarat

Abstract

This paper is a theoretical and numerical contribution to the investigation of steady two-dimensional magnetohydrodynamic (MHD) boundary layer flow of incompressible, electrically conducting nanofluid past a semi-infinite, flat plate in the presence of thermal radiation. The model considers a combination of the effects of an applied transverse magnetic field, radiative heat transfer, Brownian movement, and thermophoresis on the properties of momentum, heat, and mass transfer. Appropriate similarity transformations are made on the governing nonlinear partial differential equations to reduce them to a coupled system of nonlinear ordinary differential equations. The resultant problem of the boundary value is then resolved numerically as per the Bernoulli wavelet collocation method (BWCM). The validity of the proposed method is tested on the comparison with solutions found on the NDSolve routine of Mathematica and Haar wavelet method, and excellent agreement is achieved. A close parametric analysis shows that the magnetic fields play a significant role in suppressing the velocity of fluids and the effects of thermal radiation and nanoparticles amplify the thermal and concentration boundary layers. It is also noted that skin friction coefficient does not depend on change in Prandtl number, Brownian motion parameter, thermophoresis parameter, radiation parameter and Lewis number.

Keywords: Magnetohydrodynamic Flow, Boundary Layer Analysis, Thermal Radiation, Nanofluid, Bernoulli Wavelet Collocation Method, Heat Transfer.

Introduction

Boundary layer theory is a fundamental one in fluid mechanics and applied mathematics, because it describes the behavior of the flow of a fluid in immediate proximity to a solid surface where the viscous effects of the fluid predominate [1]. Since the development of the ideas of boundary layer, extensive research has been conducted to understand the role of various physical effects on flow, heat transfer, and mass transfer processes [2]. Among these effects, magnetohydrodynamics (MHD) and thermal radiation have gained a lot of attention because of their wide variety of industrial and technological applications.

Magnetohydrodynamics is concerned with the interaction of electrically conducting fluids and imposed magnetic fields. When a magnetic field is placed on a conducting fluid, the electromagnetic forces, called Lorentz forces, are created [3]. These forces counter the fluid

motion and alter considerably the velocity distributions, shear stress and transport characteristics. MHD flows are found in many applications including electromagnetic casting, liquid metal cooling in nuclear reactors, plasma technology, geothermal systems, metallurgical processing and magnetic drug targeting.

Thermal radiation is increasingly important in high-temperature environments where the radiative heat transfer is comparable to or more important than conductive and convective heat transfer. Examples are combustion chambers, gas turbines, nuclear reactors and aerospace systems [4]. In such cases, the neglect of radiation effect may lead to the incorrect prediction of the temperature distribution and surface heat transfer rates. Therefore, the inclusion of thermal radiation in the analysis of boundary layer has become essential to the realistic modeling of thermal systems.

In the past years, nanofluids, a suspension of nanoscale particles in traditional base fluids, have been investigated widely because of their high thermal conductivity and superior heat transfer properties [5]. The presence of nanoparticles changes the thermal and mass diffusion behaviour of the fluid. Within the field of nanofluids, Brownian motion and thermophoresis are known to be two of the major mechanisms that control the movements of nanoparticles. Brownian motion helps improve the diffusion of nanoparticles while thermophoresis makes particles migrate when there is a temperature gradient.

From the mathematical point of view, the boundary layer flows with magnetic field, thermal radiation and nanofluid effects are controlled by strongly nonlinear and coupled differential equations. Closed form analytical solutions are rarely possible, so numerical and semi-analytical techniques are indispensable [6]. Wavelet based numerical methods have become effective tools for solving nonlinear differential equations because of their flexibility, fast convergence and high accuracy. Among these the Bernoulli wavelet collocation method has shown great potential in solving nonlinear boundary value problems encountered in fluid dynamics and heat transfer [7].

The present study aims to provide the detailed boundary layer analysis of magnetohydrodynamic nanofluid flow in the presence of thermal radiation using the Bernoulli wavelets collocation method. The objectives for the study are the development of a consistent mathematical model, the application of an efficient numerical technique, the validation of the numerical results, and the study of the influence of key physical parameters on flow and transport characteristics. The presented study adds to the literature by providing a holistic analysis of MHD, radiative and nanofluid effects in a strong numerical framework.

Physical Model and Assumptions

A steady, two-dimensional, laminar boundary layer flow of an incompressible and electrically conducting nanofluid over a semi-infinite flat plate is considered. The coordinate system is chosen such that the x-axis is aligned along the flat plate in the direction of the flow, while the y-axis is taken normal to the plate [8]. The plate is maintained at a constant surface temperature T_w and a constant nanoparticle concentration C_w . Far away from the plate, the fluid attains uniform free-

stream conditions characterized by the temperature T_∞ and nanoparticle concentration C_∞ . A uniform magnetic field of strength B_0 is applied perpendicular to the plate surface, influencing the motion of the electrically conducting nanofluid.

To simplify the mathematical modeling of the problem, the following assumptions are adopted. The flow is assumed to be steady, laminar, and two-dimensional in nature. The nanofluid is treated as incompressible and Newtonian. The magnetic Reynolds number is assumed to be sufficiently small so that the induced magnetic field generated by fluid motion is negligible compared to the applied magnetic field. Effects due to viscous dissipation and Joule heating are neglected, as they are assumed to be insignificant for the present flow configuration [9]. Thermal radiation is incorporated into the energy equation using the Rosseland diffusion approximation, which is valid for optically thick fluids. The behavior of the nanofluid is described using the Buongiorno model, which accounts for nanoparticle transport due to Brownian motion and thermophoresis. Under these assumptions, the governing equations describing momentum, heat, and mass transfer within the boundary layer are formulated.

For clarity, the key physical assumptions and flow characteristics adopted in the present study are summarized in Table 1 [10].

Table 1: Summary of Physical Model and Assumptions

Sl. No.	Aspect	Description
1	Flow nature	Steady, laminar, two-dimensional boundary layer flow
2	Fluid type	Incompressible, electrically conducting Newtonian nanofluid
3	Geometry	Semi-infinite flat plate

4	Magnetic field	Uniform transverse magnetic field B_0
5	Magnetic effects	Induced magnetic field neglected (low magnetic Reynolds number)
6	Thermal effects	Thermal radiation modeled using Rosseland approximation
7	Dissipation effects	Viscous dissipation and Joule heating neglected
8	Nanofluid model	Buongiorno model with Brownian motion and thermophoresis
9	Surface conditions	Constant wall temperature T_w and concentration C_w
10	Free-stream conditions	Uniform temperature T_∞ and concentration C_∞

Mathematical Formulation

Under the boundary layer approximations and the assumptions stated above, the governing equations for mass conservation, momentum, energy, and nanoparticle concentration are expressed as follows [11].

The continuity equation is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

The momentum equation in the presence of a transverse magnetic field is written as

$$\nu \frac{\partial^2 u}{\partial y^2} - u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} - \frac{\sigma B_0^2}{\rho} (u - U_\infty) = 0 \quad (2)$$

The energy equation, which includes the effects of thermal radiation, Brownian motion, and thermophoresis [12], is given by

$$\alpha \frac{\partial^2 T}{\partial y^2} - u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} + \frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] = 0 \quad (3)$$

The nanoparticle concentration equation, accounting for Brownian diffusion and thermophoretic effects [13, 14], is expressed as

$$D_B \frac{\partial^2 C}{\partial y^2} - u \frac{\partial C}{\partial x} - v \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} = 0 \quad (4)$$

The associated boundary conditions for the flow, thermal, and concentration fields are defined as

$$u = 0, \quad v = 0, \quad T = T_w, \quad C = C_w \quad \text{at } y = 0, \quad (5a)$$

$$u \rightarrow U_\infty, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty. \quad (5b)$$

Similarity Transformation

In order to reduce the governing partial differential equations to a system of ordinary differential equations appropriate similarity transformations are introduced. These transformations are a combination of the independent variables into one similarity variable and the velocity components in terms of a dimensionless stream function. The similarity variable and velocity components are given by

$$\eta = y \sqrt{\frac{U_\infty}{\nu x}}, \quad u = U_\infty f'(\eta), \quad v = \frac{1}{2} \sqrt{\frac{\nu U_\infty}{x}} (\eta f'(\eta) - f(\eta)) \quad (6)$$

Here, $f(\eta)$ is the dimensionless stream function, and primes denote differentiation with respect to the similarity variable η [15].

The dimensionless temperature and nanoparticle concentration are introduced as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (7)$$

Substituting the similarity transformations given by equations (6) and (7) in the governing equations (1) - (4), the partial differential equations are transformed into the following system of nonlinear coupled ordinary differential equations [16].

The transformed momentum equation takes the form

$$f''' + \frac{1}{2} f f'' - M (f' - 1) = 0 \quad (8)$$

The transformed energy equation, which incorporates the effects of thermal radiation, Brownian motion, and thermophoresis, is given by

$$(1 + Rd)\theta'' + Pr \left(\frac{1}{2} f \theta' + Nb \theta' \phi' + Nt (\theta')^2 \right) = 0 \quad (9)$$

The transformed nanoparticle concentration equation is expressed as

$$\phi'' + Le \left(\frac{1}{2} f \phi' + \frac{Nt}{Nb} \theta'' \right) = 0 \quad (10)$$

The corresponding boundary conditions in terms of the similarity variables are written as

$$f(0) = 0, \quad f'(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1 \quad (11a)$$

$$f'(\infty) = 1, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0 \quad (11b)$$

The system of equations (8) - (10) along with the boundary conditions (11a) and (11b) form a nonlinear boundary value problem that governs the velocity, temperature and concentration fields in the magnetohydrodynamic boundary layer in the presence of thermal radiation [17].

Numerical Method: Bernoulli Wavelet Collocation Method

The system of nonlinear ordinary differential equations given by equations (8)–(10), together with the boundary conditions (11a)–(11b), is solved numerically using the Bernoulli wavelet collocation method. In this approach, the unknown functions $f(\eta)$, $\theta(\eta)$, and $\phi(\eta)$ are approximated as finite series expansions in terms of Bernoulli wavelet basis functions defined over the computational domain. These wavelet functions possess desirable properties such as orthogonality, smoothness, and rapid convergence, making them suitable for solving nonlinear boundary value problems [18].

Let the unknown function $g(\eta)$ represent any of the dependent variables $f(\eta)$, $\theta(\eta)$, or $\phi(\eta)$. It is approximated as

$$g(\eta) \approx \sum_{i=0}^N a_i B_i(\eta), \quad (12)$$

where $B_i(\eta)$ denotes the Bernoulli wavelet basis functions and a_i are unknown coefficients to be determined.

Substituting these approximations in the governing equations (8) - (10) and letting the resulting residuals disappear at selected collocation points leads to a system of nonlinear algebraic equations in terms of the unknown coefficients a_i . This algebraic system is solved iteratively by some suitable numerical solver until the convergence criterion is satisfied. The convergence is monitored by ensuring that successive iterations are less than prescribed tolerance [19].

To check the accuracy and reliability of Bernoulli wavelet collocation method the numerical results obtained is compared with the solutions generated using NDSolve command in Mathematica and Haar wavelet method [20]. The good agreement between these solutions establishes the validity, numerical stability and high accuracy of the proposed method for the present magnetohydrodynamic boundary layer problem.

Results and Discussion

The numerical solutions are obtained by the Bernoulli wavelet collocation method which is used to determine the effects of various physical parameters on the velocity, temperature, and nanoparticle concentration distributions, as well as engineering quantities of interest such as the coefficient of skin friction and the Nusselt number [21].

To verify the numerical accuracy of the method, the values of the velocity gradient from BWCM are compared with the values obtained using the Mathematica routine NDSolve. The comparison is given in figure 1 where an excellent agreement is shown between the two methods and very small absolute errors are shown. This validates the efficiency of Bernoulli wavelet collocation approach for the solution of present nonlinear boundary value problem.

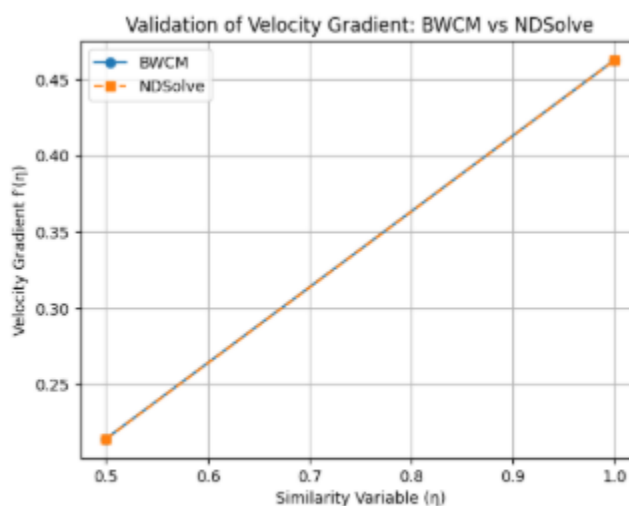


Figure 1: Validation of velocity gradient $f'(\eta)$ obtained using BWCM and NDSolve.

The effect of the magnetic parameter M on the coefficient of skin friction is shown in Figure 2. It is observed that the increase in the magnetic parameter results in increase in skin friction coefficient. This behavior is attributed to the presence of the Lorentz Force; the force implies the resistance of fluid motion and increases the shear stress at the plate surface [22].

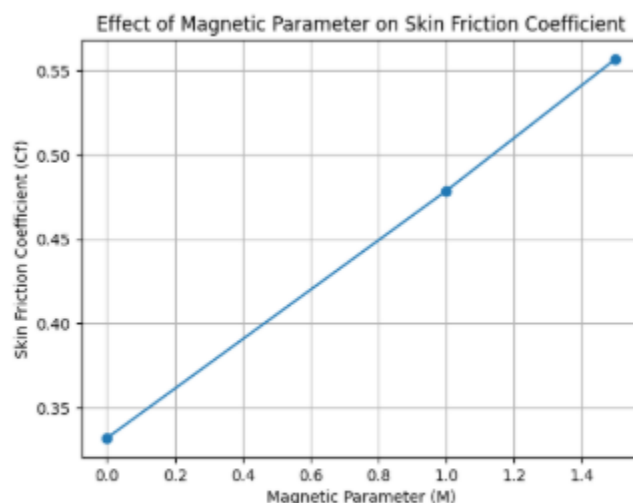


Figure 2: Variation of skin friction coefficient with magnetic parameter.

The effect of a radiation parameter R_d on the rate of heat transfer, which is represented by the Nusselt number, is shown in Figure 3. As radiation parameter increases Nusselt number decreases, so heat transfer on the surface decreases due to increased thermal diffusion due to radiative heat transfer [23].

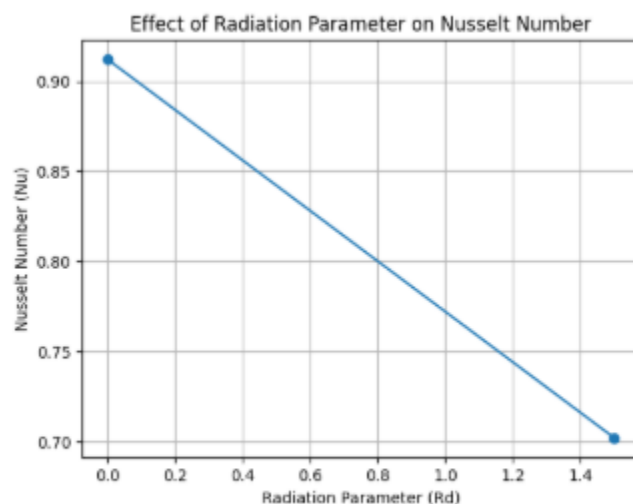


Figure 3: Variation of the Nusselt number with radiation parameter.

The obtained results show that by increasing the magnetic parameter, the fluid velocity is reduced significantly due to the opposing direction of the Lorentz force, thus thickening the momentum boundary layer [24]. An increase in the radiation parameter increases the temperature distribution within the boundary layer resulting in a thicker thermal boundary layer. Brownian motion and thermophoresis parameters are a key factor in the concentration profile modification of nanoparticles through the enhancement of the diffusion and particle migrations induced by the temperature gradients.

One of the most important results of present analysis is that the skin friction coefficient is unaffected by the variation of the Prandtl number, Brownian motion parameter, thermophoresis parameter, radiation parameter and Lewis number [25]. This observation suggests that magnetic effects rather than thermal or mass diffusion mechanisms are the main factors which govern surface shear stress in the present flow configuration.

Conclusion

The present study has given a detailed boundary layer analysis of steady 2-dimensional magnetohydrodynamic nanofluid flow over a flat plate in the presence of thermal radiation by using Bernoulli wavelet collocation method. The governing nonlinear equations were successfully transformed and solved with high numerical accuracy and the results showed excellent agreement with those obtained using established numerical solvers which confirmed the reliability of the proposed approach. The analysis showed the applied magnetic field considerably suppressed the fluid velocity by the Lorentz force causing increased wall shear stress, while the effects of thermal radiation and nanoparticles improved the temperature and concentration distribution in the boundary layer. An important result of the study is that the friction coefficient of the skin is not affected by changes in the Prandtl number, Brownian motion parameter, thermophoresis parameter, radiation parameter and Lewis number, which shows the controlling effects of the

magnetic effects on the surface shear behavior. The results represent a step further in the understanding of coupled MHD, thermal and mass transfer phenomena and an efficient and robust numerical methodology, the Bernoulli wavelet collocation method, to analyze complex boundary layer flows which are relevant for advanced thermal and engineering applications.

References

1. Abdel-Rahman, R. G., Khader, M. M., & Megahed, A. M. (2013). Melting phenomenon in magneto hydro-dynamics steady flow and heat transfer over a moving surface in the presence of thermal radiation. *Chinese Physics B*, 22(3), 030202.
2. Akinbo, B. J., & Olajuwon, B. I. (2019). Heat and mass transfer in magnetohydrodynamics (MHD) flow over a moving vertical plate with convective boundary condition in the presence of thermal radiation. *Sigma Journal of Engineering and Natural Sciences*, 37(3), 1031-1053.

3. Aly, E. H., & Sayed, H. M. (2014). Thermal radiation effects on magnetohydrodynamic boundary-layer flow due to a moving extensible surface in nanofluids. *Journal of Computational and Theoretical Nanoscience*, 11(8), 1756-1765.
4. Aly, E. H., & Sayed, H. M. (2017). Magnetohydrodynamic and thermal radiation effects on the boundary-layer flow due to a moving extensible surface with the velocity slip model: A comparative study of four nanofluids. *Journal of Magnetism and Magnetic Materials*, 422, 440-451.
5. Anuar Mohamed, M. K., Abas, F. N., & Salleh, M. Z. (2019, November). MHD boundary layer flow over a permeable flat plate in a ferrofluid with thermal radiation effect. In *Journal of Physics: conference series* (Vol. 1366, No. 1, p. 012014). IOP Publishing.
6. Arifuzzaman, S. M., Biswas, P., Mehedi, M. F. U., Al-Mamun, A., Ahmmed, S. F., & Khan, M. S. (2018). Analysis of unsteady boundary layer viscoelastic nanofluid flow through a vertical porous plate with thermal radiation and periodic magnetic field. *Journal of Nanofluids*, 7(6), 1122-1129.
7. Chaudhary, S., Singh, S., & Chaudhary, S. (2015). Thermal radiation effects on MHD boundary layer flow over an exponentially stretching surface. *Applied Mathematics*, 6(02), 295.
8. Das, K. (2014). Radiation and melting effects on MHD boundary layer flow over a moving surface. *Ain Shams Engineering Journal*, 5(4), 1207-1214.
9. Ghadikolaei, S. S., Hosseinzadeh, K., Yassari, M., Sadeghi, H., & Ganji, D. D. (2017). Boundary layer analysis of micropolar dusty fluid with TiO₂ nanoparticles in a porous medium under the effect of magnetic field and thermal radiation over a stretching sheet. *Journal of Molecular Liquids*, 244, 374-389.
10. Hayat, T., Muhammad, T., Alsaedi, A., & Alhuthali, M. S. (2015). Magnetohydrodynamic three-dimensional flow of viscoelastic nanofluid in the presence of nonlinear thermal radiation. *Journal of Magnetism and Magnetic Materials*, 385, 222-229.
11. Hayat, T., Shafiq, A., & Alsaedi, A. (2016). Hydromagnetic boundary layer flow of Williamson fluid in the presence of thermal radiation and Ohmic dissipation. *Alexandria Engineering Journal*, 55(3), 2229-2240.
12. Hussain, S. M., Sharma, R., Seth, G. S., & Mishra, M. R. (2018). Thermal radiation impact on boundary layer dissipative flow of magneto-nanofluid over an exponentially stretching sheet. *Int. J. Heat Technol*, 36(4), 1163-1173.
13. Khan, M., Hussain, M., & Azam, M. (2016). Magnetohydrodynamic flow of Carreau fluid over a convectively heated surface in the presence of non-linear radiation. *Journal of magnetism and magnetic materials*, 412, 63-68.
14. Madhu, M., Kishan, N., & Chamkha, A. J. (2017). Unsteady flow of a Maxwell nanofluid over a stretching surface in the presence of magnetohydrodynamic and thermal radiation effects. *Propulsion and Power research*, 6(1), 31-40.

15. Makinde, O. D. (2011). Second law analysis for variable viscosity hydromagnetic boundary layer flow with thermal radiation and Newtonian heating. *Entropy*, 13(8), 1446-1464.
16. Megahed, A. M. (2014). Variable heat flux effect on magnetohydrodynamic flow and heat transfer over an unsteady stretching sheet in the presence of thermal radiation. *Canadian Journal of Physics*, 92(1), 86-91.
17. Nadeem, S., & Haq, R. U. (2014). Effect of thermal radiation for magnetohydrodynamic boundary layer flow of a nanofluid past a stretching sheet with convective boundary conditions. *Journal of Computational and Theoretical Nanoscience*, 11(1), 32-40.
18. Nandeppanavar, M. M., Vajravelu, K., & Abel, M. S. (2011). Heat transfer in MHD viscoelastic boundary layer flow over a stretching sheet with thermal radiation and non-uniform heat source/sink. *Communications in Nonlinear Science and Numerical Simulation*, 16(9), 3578-3590.
19. Pal, D., & Mandal, G. (2017). Thermal radiation and MHD effects on boundary layer flow of micropolar nanofluid past a stretching sheet with non-uniform heat source/sink. *International Journal of Mechanical Sciences*, 126, 308-318.
20. Pal, D., & Talukdar, B. (2010). Perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. *Communications in Nonlinear Science and Numerical Simulation*, 15(7), 1813-1830.
21. Reddy, M. G. (2014). Influence of magnetohydrodynamic and thermal radiation boundary layer flow of a nanofluid past a stretching sheet. *Journal of Scientific Research*, 6(2), 257-272.
22. Sedighi, A. A. (2017). Steady boundary layer magnetohydrodynamic viscous flow and heat transfer of nanofluid over stretching sheet in presence of radiation and heat source. *Journal of Nanofluids*, 6(6), 1206-1214.
23. Shateyi, S., & Prakash, J. (2014). A new numerical approach for MHD laminar boundary layer flow and heat transfer of nanofluids over a moving surface in the presence of thermal radiation. *Boundary value problems*, 2014(1), 2.
24. Siddiqua, S., Hossain, M. A., & Gorla, R. S. R. (2012). Conduction-radiation effects on periodic magnetohydrodynamic natural convection boundary layer flow along a vertical surface. *International journal of thermal sciences*, 53, 119-129.
25. Sivaiah, G., Reddy, K. J., Reddy, P. C., & Raju, M. C. (2019). Numerical study of mhd boundary layer flow of a viscoelastic and dissipative fluid past a porous plate in the presence of thermal radiation. *International Journal of Fluid Mechanics Research*, 46(1).